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A black hole hologram in de Sitter space

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Abstract: In this paper we show that the entropy of de Sitter space with a black hole in arbitrary dimension can be understood using a modified Cardy-Verlinde entropy formula. We also comment on the observer dependence of the de Sitter entropy.

Keywords: Black Holes in String Theory, AdS/CFT Correspondence, Cosmology of Theories beyond the SM.

Contents

1. Introduction

Physics is very different depending on the presence and sign of the cosmological constant. With a vanishing cosmological constant, space time is asymptotically flat and physics can be conveniently described by an S-matrix. This is a consequence of the presence of a light like infinity; if we wait long enough particles may be separated by an arbitrary spatial distance and interactions will be suppressed. In the case of a negative cosmological constant the universe is anti de Sitter (AdS) on large scales and there is no light like infinity. There is, however, at an infinite spatial distance a time like infinity which can be used as a holographic screen of lorentzian signature, [1, 2, 3]. This discovery was the first rigorous implementation of the idea of holography, [4, 5], and has lead to many subsequent studies.

But it is very likely that our universe is neither flat nor AdS but instead is a de Sitter space (dS) on large scales with a positive cosmological constant, [6, 7]. While being the most realistic possibility, de Sitter space is at the same time theoretically the most challenging. To this date there is, in fact, no successful implementation of de Sitter space in string theory.

One of the intriguing features of de Sitter space is the presence of a horizon and an associated temperature and entropy, [8]. While string theory successfully has addressed the problem of entropy for black holes, dS entropy remains a mystery. One reason is that the finite entropy seems to suggest that the Hilbert space of quantum gravity for asymptotically de Sitter space is finite dimensional, [9]. A recent discussion of this and other connected issues can be found in [10]. Another, related, reason is that the horizon and entropy in de Sitter space have an obvious observer dependence. For a black hole in flat space (or even in AdS) we can take the point of view of an outside observer who can assign a unique entropy to the black hole. The problem of what an observer venturing inside the black hole experiences, is much more tricky and has not been given a satisfactory answer within string theory. While the idea of black hole complementarity provides useful clues, [11], rigorous calculations are still limited to the perspective of the outside observer. In de Sitter space there is no way to escape the problem of the observer dependent entropy. This contributes to the difficulty of de Sitter space.

Recently there has been some progress towards a holographic understanding of de Sitter space, [10, 12]. See also [13, 14, 15]. A review with references may be found in [16]. The main observation is that there is a possibility of introducing a holographic screen at either time like past infinity, $\mathcal{I}^-$, or time like future infinity, $\mathcal{I}^+$. The theory on the screen will, just as in the case of AdS, be a conformal field theory with a scale that encodes the dimension transverse to the screen. But in contrast to the AdS case the holographic theory of de Sitter space will be euclidean and it is time that is given a description through the scale. Large scales on $\mathcal{I}^+$ will correspond to early times, while late times will correspond to small scales. The central charge of the theory is given by the area of the horizon. Very recently it was suggested, [17], (see also [18]) that our universe is described by a RG flow in the euclidean theory on $\mathcal{I}^+$. In the IR the theory has a fixed point of relatively low central charge corresponding to an early phase of inflation with a horizon a few orders of magnitude larger than the Planck scale. In the UV, on the other hand, there is a fixed point of large central charge corresponding to the universe we now are approaching where a cosmological constant again will be dominating.

In this paper we will continue the study of a holographic description of de Sitter space. In particular we will investigate the properties of black hole holograms in de Sitter space. We will show that the entropy of the cosmological horizon in the presence of a black hole can be understood using a Cardy-Verlinde formula, [19], in a way very similar to the entropy of black holes in AdS. We will also make some speculations on how to address the problem of observer dependence.

2. Constructing the hologram

2.1 The metric

To construct the hologram we must find appropriate coordinates to use when we approach the space like boundary. We will concentrate on $\mathcal{I}^+$ which seems to be the most reasonable choice in a realistic cosmology. While our universe is expected to become exactly de Sitter in the future, there may very well have been a messy beginning before the inflationary de Sitter space in the early universe which precludes the existence of a well defined $\mathcal{I}^-$. Clearly there are many possible coordinate systems to choose among, but we will concentrate on two important and interesting possibilities: static and cosmological coordinates, respectively. Both can be used to approach $\mathcal{I}^+$ and will lead to different coordinates for the boundary

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theory. The static choice will lead to what we will denote as cylindrical coordinates, while the cosmological choice will lead to planar coordinates. We will now consider these two possibilities in turn.

2.1.1 Cylindrical coordinates

The d-dimensional de Sitter black hole in static coordinates is given by

$$ds^2 = -V dt^2 + V^{-1} dr^2 + r^2 d\Omega_{d-2}^2 \,, \tag{2.1}$$

where $V = 1 - \frac{r^2}{R^2} - \frac{2M}{r^{d-3}}$. Throughout the paper we will put $G = 1$ where G is Newtons constant. The advantage of these coordinates is their obvious simplicity and time independence. The disadvantage is that the expansion of the universe is not manifest. As is clearly seen there are in general two kinds of horizons, one associated with the black hole and one cosmological. $\mathcal{I}^+$,which is in the focus of our interest, is located outside the future cosmological horizon where r is time like and t space like. For $r \gg R$ the metric becomes

$$ds^2 = -\frac{R^2}{r^2} dr^2 + \frac{r^2}{R^2}\left(dt^2 + R^2 d\Omega_{d-2}^2\right).$$

It is clear that $\mathcal{I}^+$ is approached for large r and we note that the boundary is mapped on to an euclidean cylinder $\mathbb{R} \times S^{d-2}$ where t is the coordinate along the cylinder. This kind of coordinates were used in [18].

2.1.2 Planar coordinates

Another convenient coordinate system is obtained by introducing

$$\begin{cases} \rho = \rho\,(t,r) = r e^{-\tau/R} \,, \\ \tau = \tau\,(t,r) = t + \frac{R}{2}\ln\left(\frac{r^2}{R^2} - 1\right). \end{cases} \tag{2.2}$$

With no black hole this becomes an inflating cosmology according to

$$-d\tau^2 + e^{2\tau/R}\left(d\rho^2 + \rho^2 d\Omega_{d-2}^2\right),$$

i.e. the cosmological form of the de Sitter space. These coordinates have the obvious advantage of making the expansion of the universe explicit with a Hubble constant given by $1/R$. Without a black hole the geodesics are simply given by constant comoving spatial coordinates. One should note, however, that this metric only covers half of the space time. (Another half, bounded by $\mathcal{I}^-$, is covered by changing $e^{2\tau/R}$ to $e^{-2\tau/R}$.) $\mathcal{I}^+$ is now approached for large τ and it is clear that it will be described by spherical coordinates on the plane. Applying the same coordinate change to the metric with a black hole leads to the slightly more complicated metric

$$ds^2 = -N_\tau^2 d\tau^2 + h\left(d\rho + N_\Sigma d\tau\right)^2 + r^2 d\Omega_{d-2}^2 \,, \tag{2.3}$$

where, with $V_0 = 1 - \frac{r^2}{R^2}$, we have

$$h = \frac{r^2}{V}\left(1 - \frac{r^2 V^2}{R^2 V_0^2}\right)\rho^{-2}\,, \qquad N_\Sigma = \frac{1 - \frac{V^2}{V_0^2}}{1 - \frac{r^2V^2}{R^2V_0^2}}\frac{\rho}{R}\,, \qquad \text{and} \qquad N_\tau = \frac{\sqrt{V}}{\sqrt{1 - \frac{r^2V^2}{R^2V_0^2}}}\,.$$

If we fix r and make a translation in t we see from (2.2) that this corresponds to a scaling of ρ that leaves the metric invariant even in the presence of the black hole. The crucial property that makes this possible is that the metric can be written in the static, time independent form above. Note however, that the corresponding Killing vector is not globally time like.

2.2 The Brown-York tensor

We have now obtained coordinates which can be used to describe the holographic theory. We have also seen the relation between conformal transformations in the hologram and time translations in the de Sitter space. To proceed we must find out more about the generators of the transformations and their corresponding conserved quantities. To this end, let us follow the recipe of [18, 20] and calculate the Brown-York tensor, [21], of the boundary which is the generator of coordinate changes in the bulk. The subtracted Brown-York tensor is given by

$$T_{ij} = -\frac{1}{8\pi}\left(K_{ij} - Kh_{ij} - \frac{d-2}{R}h_{ij} - \frac{R}{d-3}G_{ij} + \cdots\right),$$

where the last term is lacking in $d = 3$, and the $\cdots$ refers to terms needed when $d > 5$. The terms added to the $K_{ij} - Kh_{ij}$ are counter terms in the boundary theory needed to render T_{ij} finite. We have that

$$K_{ij} = -\frac{1}{2}\left(h_{il}g^{l\mu}\nabla_\mu u_j + h_{jl}g^{l\mu}\nabla_\mu u_i\right),$$

where u^μ is a forward pointing, time like, unit normal to surfaces of constant τ. $g_{\mu\nu}$ is the metric of the full space time while h_{ij} is the piece induced on the boundary. The conserved quantity associated with the bulk Killing vector is given by

$$Q = \oint_\Sigma d^{d-2}\Omega\sqrt{h}n^i\xi^j T_{ij}\,,$$

where n^i is an outward pointing unit normal to surfaces of constant ρ and ξ^j is the Killing vector. Note that ξ^j is space like near the boundary. As argued in [18] one can choose coordinates in such a way that ξ^j is proportional to n^i with the constant of proportionality given by the appropriate lapse function.[1]

Let us now calculate the Brown-York tensor in the two coordinate systems that we have chosen.

2.2.1 Planar coordinates

The calculation in planar coordinates involves the non-diagonal metric (2.3). A straight forward calculation gives, in particular,

$$T_{\rho\rho} = -\frac{1}{8\pi}(d-2)\frac{M}{r^{d-3}}\frac{1}{\rho^2}$$

[1]Note that this choice is, in general, not possible for other conserved quantities like angular momentum.

at large τ. In this case the $G_{ij} + \cdots$ counter terms vanish as $\tau \to \infty$ and we expect the above result to be true for arbitrary dimension. To obtain the conserved charge we use $\xi^j = \frac{r}{\sqrt{h}} n^i$, where the coefficient is the proper lapse function to make sure that our conserved quantity is conjugate to translations in t. After some calculations we find that

$$Q = -\frac{d-2}{8\pi}\Omega_{d-2}M\,,$$

where $\Omega_{d-2} = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})}$ is the surface area of the unit $d-1$ sphere.

But how do we make contact with the CFT? From (2.2) it follows (for fixed r) that

$$\rho\frac{d}{d\rho} = -R\frac{d}{dt}\,,$$

and we therefore conclude that the energy E of the CFT, conjugate to ρ, is given by

$$E = -\frac{d-2}{8\pi}\Omega_{d-2}\frac{MR}{\rho}\,, \tag{2.4}$$

in planar coordinates. The sign follows provided that the conformal generator acts in the direction of decreasing ρ, (i.e. increasing t). Let us now proceed to perform the corresponding calculation in cylindrical coordinates.

2.2.2 Cylindrical coordinates

In this case the metric is a lot simpler than in the planar case, and is just given by (2.1). The result of the calculation is of the form

$$E = E_{C,A} - \frac{d-2}{8\pi}\Omega_{d-2}M\,,$$

where $E_{C,A}$ corresponds to an anomalous Casimir contribution. Note that the energy E is a dimensionful quantity which transforms in a non-trivial way when we go between the cylinder and the plane, even if we forget about the anomaly. In cylindrical coordinates it is independent of the direction t along the cylinder, while on the plane it has the radial dependence given by eq. (2.4). $E_{C,A}$ vanishes for even d, while we have

$$\begin{aligned} E_{C,A} &= \frac{1}{8}\,, \qquad \text{for } d=3\,,\\ E_{C,A} &= \frac{3\pi R^2}{32}\,, \qquad \text{for } d=5\,. \end{aligned}$$

The appropriate expressions for higher dimensions need the precise form of the higher order counter terms. Similar formulae can also be obtained in the case of AdS space. The only differences are that the M dependent term comes with a positive sign in all dimensions, while the anomaly is negative in $d=3$.[2]

[2] The signs are quite subtle in the case of de Sitter. Our signs agree with those of [18] but are different from those of [27] which focused on $\mathcal{I}^-$. For more comments on this, see the last footnote of the paper.

3. Interpreting the hologram

We have now completed the first steps towards constructing the black hole hologram. In order to investigate it further we need an additional tool in the form of the Cardy-Verlinde entropy formula, [19]. We will begin by recalling how the formula is applied to the case of AdS before we proceed with the generalization to de Sitter.

3.1 Black holes in AdS

In [19] it was shown that a strongly coupled CFT (with an AdS-dual) at temperature T on a cylinder $\mathbb{R} \times S^{d-2}$ (where S^{d-2} has constant radius R) with a central charge c,[3] has an energy given by

$$E = \frac{(d-2)\,c}{48\pi} \frac{V}{L^{d-1}} \left(1 + \frac{L^2}{R^2}\right) \equiv E_E + E_C\,. \tag{3.1}$$

where $V = \Omega_{d-2} R^{d-2}$ is the volume of the S^{d-2}.[4] The energy is the sum of an extensive contribution E_E and an intensive Casimir contribution $E_C > 0$.[5] L is a parameter related to the temperature given by

$$T = \frac{1}{4\pi L} \left(d - 1 + (d-3)\,\frac{L^2}{R^2}\right),$$

and the entropy, finally, is given by

$$S = \frac{c}{12} \frac{V}{L^{d-2}} = \frac{4\pi R}{d-2} \sqrt{E_C E_E}\,.$$

The form was argued on general grounds, while the detailed coefficients were deduced from the AdS/CFT correspondence applied to black holes in AdS. We will verify this in detail below after generalizing the expressions to de Sitter.

Let us now investigate what is going on a little bit more carefully. Depending on the coordinates we use, the energy of the state we are considering will be different. If we use planar coordinates on the boundary,[6] we find that the AdS space has vanishing energy, while a black hole in AdS will have positive energy. It is useful to write this energy as

$$E = \frac{E_{\text{plan}} R}{\rho},$$

where E_{plan} is a rescaled energy given by

$$E_{\text{plan}} = \frac{d-2}{8\pi} \Omega_{d-2} M\,.$$

[3] Following [19] we define the central charge in terms of the subextensive Casimir energy (including possible temperature dependent terms). Since the Casimir energy is proportional to the number of massless fields in the CFT, the same will be true for the central charge.

[4] Another possibility is to choose the radius of the cylinder to be the horizon radius r_s rather than R. This modifies the subsequent calculations but leads to the same result.

[5] Our convention for E_C differs by a factor of 2 from [19].

[6] This corresponds, basically, to Poincare coordinates in the bulk. For more details on the choice of vacua in the case of AdS_3 see [22].

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If we instead use cylindrical coordinates, we find another vacuum where the AdS space has a shifted energy due to the anomaly (if d is odd), and we have that

$$E_{\text{cyl}} = \frac{d-2}{8\pi}\Omega_{d-2}M + E_{C,A}\,,$$

where $E_{C,A}$ is the anomalous contribution to the Casimir energy. (Note that $E_{C,A}$ is negative for $d = 3$ and positive for $d = 5$.) The analysis of [19] was performed in cylindrical coordinates based on the work of [23, 24]. There the expressions for e.g. the action in the presence of a black hole had been found to be divergent and was rendered finite by subtracting the equally divergent contribution from empty AdS. It is therefore the energy in excess of the vacuum associated with empty AdS space (with an extensive contribution as well as a Casimir contribution) that is given by eq. (3.1). Hence we conclude that

$$E_E + E_C = \frac{d-2}{8\pi}\Omega_{d-2}M\,,$$

and the entropy is given by

$$S = \frac{4\pi R}{d-2}\sqrt{E_C\left(E_{\text{plan}} - E_C\right)}\,.$$

This is the generalization of the Cardy formula proposed in [19]. Note that the total energy in cylindrical coordinates is given by $E_{\text{cyl}} = E_E + E_C + E_{C,A}$. For $d = 3$ one has $E_C = -E_{C,A} > 0$ and it follows that $E_{\text{cyl}} = E_E$.

3.2 Black holes in de Sitter

We will now proceed to the case of de Sitter space to see whether similar formulae are applicable to the relevant conjectured euclidean CFT's.[7] According to our previous calculations the de Sitter black hole is a state with negative energy. For convenience we define

$$E_{\text{plan}} = -\frac{d-2}{8\pi}\Omega_{d-2}M\,.$$

Using cylindrical coordinates, the new de Sitter vacuum has a shifted energy due to the anomaly (if d is odd) given by

$$E_{\text{cyl}} = E_{C,A} - \frac{d-2}{8\pi}\Omega_{d-2}M\,.$$

As was explained above, the anomaly $E_{C,A}$ is positive and hence has the opposite sign to AdS in the case of $d = 3$, while the sign is the same in $d = 5$. If we now subtract the contribution from empty dS, just as in the AdS case, we find

$$E_E + E_C = -\frac{d-2}{8\pi}\Omega_{d-2}M\,.$$

To be able to account for the entropy in de Sitter space we conclude, following [19], that the Casimir contribution is given by

$$E_C = -\frac{(d-2)\,c}{48\pi}\frac{V}{L^{d-3}R^2}\,,$$

[7] For other recent attempts in this direction, see [18, 25, 26].

which is *negative* and therefore has the *opposite* sign compared to the CFT relevant for AdS. This is also what to be expected from the naive continuation $R^2 \to -R^2$. The entropy is again given by

$$S = \frac{4\pi R}{d-2}\sqrt{|E_C|\,(E_{\rm plan} - E_C)},$$

while the temperature becomes

$$T = \frac{1}{4\pi L}\left(d - 1 - (d-3)\,\frac{L^2}{R^2}\right), \tag{3.2}$$

where again R^2 has been replace by $-R^2$. This has important consequences that we will come back to shortly. As in the case of AdS one may note that $E_C = -E_{C,A} < 0$ for $d = 3$ implying $E_{\rm cyl} = E_E$.

Let us now verify that the above indeed reproduces the correct result for black holes. We will do the AdS and the de Sitter cases at the same time. The prescription tells us to impose

$$\frac{(d-2)\,c}{48\pi}\frac{V}{L^{d-1}} \pm \frac{(d-2)\,c}{48\pi}\frac{V}{R^2 L^{d-3}} = \pm\frac{d-2}{8\pi}\Omega_{d-2}M\,,$$

where the + sign refers to AdS and the — sign refers to de Sitter. With $L = R^2/r_s$ (where r_s is the horizon radius) and

$$c = 3R^{d-2}\,,$$

the equation becomes

$$1 \pm \frac{r_s^2}{R^2} - \frac{2M}{r_s^{d-3}} = 0\,, \tag{3.3}$$

which is precisely the equation for the position of a horizon. From the formula for the entropy we find

$$S = \frac{c}{12}\frac{V}{L^{d-2}} = \frac{1}{4}\Omega_{d-2}r_s^{d-2} = \frac{1}{4}A\,,$$

and hence the generalized Cardy-Verlinde formula can account for the black hole entropy in AdS as well as de Sitter space. The case $d = 3$ gives, using $E_{C,A} = c/24R$,

$$S = 4\pi R\sqrt{E_{C,A}E_E} = 2\pi\sqrt{\frac{c}{6}\left(RE_{\rm plan} + \frac{c}{24}\right)}\,,$$

where $E_{\rm plan} < 0$. In the case of AdS one has $E_{\rm plan} > 0$ and the shift is by $-c/24$.[8]

There is a slight puzzle with the above construction. In the case of de Sitter space there are two solutions of eq. (3.3), since there are two horizons. One corresponds to the cosmological de Sitter horizon, while the other corresponds to the horizon of the black hole. It is easily checked, however, that it is only the cosmological horizon that leads to a positive temperature. The conclusion, then, is that the CFT that we are studying on $\mathcal{I}^+$ is accounting for the entropy of the cosmological horizon only. This is consistent with the fact that the energy and the entropy of the system is decreased when the mass of the black hole is

[8]Note that we are considering left and right movers together with $c = c_L + c_R$.

increased.[9] For more on the entropy of black holes in de Sitter space see [28]. The total entropy of both horizons as well as the entropy of the cosmological horizon have this property, but our conclusion is that it is only the entropy of the cosmological horizon that is accounted for by the euclidean CFT on $\mathcal{I}^+$. Empty de Sitter space is also covered by the analysis and entails the presence of a gas at non-zero temperature that can carry the entropy.

3.2.1 The Nariai black hole

The largest possible black hole in de Sitter space, the Nariai black hole [29], corresponds to a situation where the two horizons coincide. It is easy to check that this occurs when

$$M = \frac{1}{d-1}\left(\frac{d-3}{d-1}\right)^{\frac{d-3}{2}} R^{d-3}\,,$$

which corresponds to zero temperature. Furthermore, on $\mathcal{I}^+$, where the energy of a black hole is lower than that of empty de Sitter space, the Nariai black hole corresponds to a lowest energy state. It would be interesting to understand this better from the point of view of the holographic theory. As observed in [18, 27] the energy in cylindrical coordinates vanishes for the Nariai black hole in $d = 5$, and a corresponding statement can also be made in $d = 3$. Is this just a coincidence or is there a deeper explanation? This should be investigated further by comparing with higher dimensional cases following, e.g. the analysis of [30, 31]. For other work in this context see e.g. [32, 33].

4. Conclusions

In this paper we have found that a modified Cardy-Verlinde entropy formula reproduces the entropy of the cosmological de Sitter horizon in the presence of a black hole. But there are several unclear points. Why does the CFT on $\mathcal{I}^+$ only provide the entropy for the cosmological horizon? Is it correct to discard the negative temperature solution? Is the role of $\mathcal{I}^-$ to take care of the black hole horizon?

Furthermore, the de Sitter horizon is an observer dependent construction. How does the CFT take this into account? Our analysis gives a hint. Let us consider a black hole at the origin of static coordinates. In planar coordinates the energy density in the boundary theory corresponding to such a black hole goes like $1/\rho^2$.[10] This is left invariant under a time translation in the bulk in the same way as the metric. If we now take the point of view of an observer who sees the black hole displaced from the center of de Sitter space, the situation is different. In the case of a black hole with a horizon much smaller than the cosmological horizon, and an observer far away from the black hole, one can (for our purposes) approximate the system with a black hole and an observer in free fall in de

[9] One may note that if the expression for the energy of the system (and therefore also the temperature) is considered to have the opposite sign, the reasoning is reversed. Positive temperature now suggests that it is the entropy of the black hole horizon that is accounted for by the theory. This is presumably the case for $\mathcal{I}^-$.

[10] Note that the subtracted Brown-York tensor is related to the energy-momentum tensor of the CFT by an infinite conformal transformation. See, e.g. [34].

Sitter space. This means that we can assume the distance between the two to be constant in comoving coordinates. As time goes by the expansion will increase the proper distance between the observer and the black hole. For the bulk observer time translational invariance will be broken and she will see how the black hole is approaching the cosmological horizon. As the black hole disappears from view, the area of the cosmological horizon (according to our observer) will increase with a net increase in entropy as a consequence. In the boundary there is a similar story. The energy density is no longer left invariant since the position of the maximum is offset from $\rho = 0$ and will effectively move off towards infinity as time goes by and we are zooming in on smaller scales. In fact, for late times the relevant physics will be taking place on small scales asymptotically independent of the black hole, and energy as well as entropy will approach the values for empty de Sitter space. It would be interesting to further study the time evolution of entropy in this setting.

In [17] it was indicated how a RG flow with a changing central charge may take us from an inflationary era in the past with a small R to a de Sitter space in the future with a large R. But even with a constant number of degrees of freedom it is interesting to study how non-trivial time evolution is encoded in various scales. What is the role of the second law in the dS/CFT correspondence? How is the Hawking evaporation of the black hole taken into account? There are clearly many interesting and important questions that need to be investigated.

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